Portfolio Picker

Technical Guide

Quintessa

Portfolio Picker 1.0.0.5 Technical Guide

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www.quintessa.org
Quintessa Limited
The Hub
14 Station Road
Henley-on-Thames
Oxfordshire
RG9 1AY
United Kingdom

Tel: +44 (0)1491 636246

https://www.quintessa.org/software/downloads-and-demos/portfolio-picker.html

Summary

Portfolio Picker is a generic portfolio selection tool that uses basic decision theory to compute an optimal selection amongst several choices when there are potentially multiple factors contributing to the outcome. This document contains technical details about the theory behind version 1.0 of Portfolio Picker, including the algorithm used to compute the optimal selections.

Contents

1 Introduction

Portfolio Picker is a generic portfolio selection tool that uses basic decision theory to compute an optimal selection amongst several choices when there are potentially multiple factors contributing to the outcome. This document contains technical details about the theory behind version 1.0 of Portfolio Picker, including the algorithm used to compute the optimal selections.

While Portfolio Picker has been developed so that it can be used without any technical information regarding the mathematics used to develop it, this document provides mathematical background for those interested in knowing more.

For examples and ideas about the applications of Portfolio Picker, see the Portfolio Picker version 1.0 Case Studies document, available from the Help menu in the application.

Section 2 details the very fundamentals of Portfolio Picker, whilst Section 3 goes into more mathematical detail. A glossary of terms used throughout this document can be found in Appendix A.

2 The Fundamentals of Portfolio Picker

Portfolio Picker uses an optimisation algorithm in order to maximise the utility of a selection. The utility of a portfolio is a mathematical concept that stems from decision theory, and depends on the information available for the selection. In terms of the basic selection process, there are four essential types of input that the algorithm needs in order to calculate a (useful) selection, as follows:

- **Categories**: These represent groups of items from which the selection will be made up. They have an associated **Number Available** that tells the algorithm how many items are available in that category to choose from;
- **Criteria**: These represent the relevant information that each category is **scored** on (see below). Criteria have an associated **Importance Value** that determines the relative importance of that criterion when compared with the other criteria. Criteria that have a higher importance value will therefore contribute more to the utility than those criteria that have a lower importance value.
- **Scores for category-criterion pairs**: Each category is scored for each criterion that has been specified. Because the algorithm acts to maximise utility, high scores for categories contribute more to the utility than lower scores. It is important that categories are scored appropriately, so 'risk-based' or 'negative' criteria should have low scores if the risk is high to reduce their contribution to the utility (for example, if the cost is high then the score for that option should be low);
- **Number to be chosen**: This tells the algorithm how many items to include in the selection.

In Portfolio Picker, items in a category are *indistinguishable*, that is, if there are two items available in category *A* picking the first one is exactly the same as picking the second one. Criteria and scoring have a level of subjective input by their very nature. Criteria can be defined from known sources or results, but also from the users own judgement on what is important to the decision being made. Scoring can be difficult, but a good approach involves choosing an appropriate scale for each criterion and being consistent with the scoring process.

Balancing strategies help provide a more balanced selection, and are discussed more in Section 3.

3 The Mathematics behind Portfolio Picker

This section details the mathematics that the algorithm used in Portfolio Picker is based on.

In general, if there are N (a user-defined, fixed value that does not change for a particular portfolio) objects to be chosen from C different categories of object, and there are E different criteria then the overall (raw) utility function U_o for the portfolio can be written:

$$U_o = \sum_{e=1}^{E} I_e U_e$$

$$U_e = \sum_{c=1}^{C} N_c S_{ce}$$

$$N = \sum_{c=1}^{C} N_c$$

where I_e ($> 0\ \forall e$) is the importance level for criterion e, S_{ce} is the score for category c and criterion e, and N_c is the number of objects selected from category c. The idea is to then select N_c to maximise the overall utility. For every possible selection of objects constituting a portfolio, a utility value is calculated. The portfolio(s) with the highest utility is(are) then the optimal choice. Portfolio Picker evaluates a utility value for each possible selection, and the selection with the largest utility is the one reported. When there are multiple 'optimal' selections, Portfolio Picker displays the one with the most objects in 'earlier' categories (as defined by the order the categories are given within the software).

To take into account the selection as a whole, a balance penalty function, P, can be introduced into the utility function to give a final utility,

$$U = U_o P$$

where P can take a number of forms ($P = 1$ if there is no balance penalty). This is only one approach amongst many, and there is no generally accepted method that can be applied across a wide range of applications (Weistroffer and Smith, 2005). The two-stage approach taken in Portfolio Picker has some similarities to the problems considered in Polyashuk (2005).

A range of different possibilities for P are available. The "Minimum Utility balancing strategy" uses a Heaviside function:

$$P = \prod_{e=1}^{E} H(U_e - U_e^{\min})$$

$$U_e^{\min} = I_e \frac{U_0}{(\sum_{e=1}^{E} I_e)^2}.$$

Selections that give insufficient consideration to one or more criteria are considered inadmissible and so excluded from the optimisation.

A smoother balance function, the "Continuous Penalty Function balancing strategy", has the following form:

$$P = G(P_e)$$

$$P_e = F\left(\frac{U_e}{U_e^t}\right)$$

$$U_e^t = I_e \frac{U_o}{\sum_{e=1}^{E} {I_e}^2}$$

The choices for F and G (where α determines how rapidly F drops away from unity) are,

$$F(x) = \min(1, x^\alpha)$$

$$G = \prod_{e=1}^{E} P_e \qquad \text{or} \qquad G = \min_e(P_e)\,.$$

When $\alpha = 1$ we obtain a linear ramp, but $\alpha < 1$ is also allowed. The two options for G provide two alternative balancing functions, namely a product ramp and a minimum ramp function (respectively).

References

Polyashuk M V (2005). A Formulation of Portfolio Selection Problem with Multiple Criteria. J. Multi-Crit. Decis. Anal. 13, 135-145.

Weistroffer H R and Smith C H (2005). Decision Support for Portfolio Problems. Proceedings of the 2005 Southern Association of Information Systems Conference.

Appendix A – Glossary of Terms

A glossary of terms used throughout this document is given in Table 1.

Table 1 Glossary of Technical Terms

Term	Description
Object	A single item, from a single category, to be included in the portfolio. An object cannot be in more than one category. Objects cannot overlap (i.e. they cannot have more than one property in version 1 of the software).
Category	A group of objects with a certain property. Categories must be given a number of objects available to choose from.
Criterion	A type of information of interest.
Portfolio/Selection	A selection of objects that gives an overall utility.
Utility function	A mathematical concept that determines the amount of 'information' that can be obtained from a given selection.
Raw utility	The value of the utility function before any penalties or optimisations have been applied.
Penalty function	Used as a multiplier to the raw utility to balance the selection.
Importance	A value linked to each criterion that signifies the relative importance that criterion has for the selection.
Score	A value linked to each category-criterion pair that indicates how well each category meets that criterion.

Term	Description
Sensitivity study	Plots the effect of changing the relative importance values for each criterion.
Optimisation	The process of improving the raw utility to provide a more balanced selection.