

Portfolio Picker

Technical Guide



Quintessa

Portfolio Picker 1.1.0.3 Technical Guide

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Summary

Portfolio Picker is a generic portfolio selection tool that uses basic decision theory to compute an optimal selection amongst several choices when there are potentially multiple factors contributing to the outcome. This document contains technical details about the theory behind version 1.1 of Portfolio Picker, including the algorithm used to compute the optimal selections.

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1 Introduction

Portfolio Picker is a generic portfolio selection tool that uses basic decision theory to compute an optimal selection amongst several choices when there are potentially multiple factors contributing to the outcome. This document contains technical details about the theory behind version 1.1 of Portfolio Picker, including the algorithm used to compute the optimal selections.

While Portfolio Picker has been developed so that it can be used without any technical information regarding the mathematics used to develop it, this document provides mathematical background for those interested in knowing more about Portfolio Picker's main functionality.



For examples and ideas about the applications of Portfolio Picker, see the Portfolio Picker Case Studies document, available from the Help menu in the application.

Section 2 contains the very fundamentals of Portfolio Picker, whilst Section 3 goes into more mathematical detail. A glossary of terms used throughout this document can be found in Appendix A.

2 The Fundamentals of Portfolio Picker

Portfolio Picker uses an optimisation algorithm in order to maximise the utility of a selection. The utility of a portfolio is a mathematical concept that stems from decision theory, and depends on the information available for the selection. In terms of the basic selection process, there are four essential pieces of information that the algorithm needs in order to calculate a (useful) selection, as follows:

- ▲ **Categories:** These represent groups of items from which the selection will be made up. They have an associated **Number Available** that tells the algorithm how many items are available in that category to choose from;
- ▲ **Criteria:** These represent the relevant information that each category is **scored** on (see below). Criteria have an associated **Importance Value** that determines the relative importance of that criterion when compared with the other criteria. Criteria that have a higher importance value will therefore contribute more to the utility than those criteria that have a lower importance value.
- ▲ **Scores for category-criterion pairs:** Each category is scored for each criterion that has been specified. Because the algorithm acts to maximise utility, high scores for categories contribute more to the utility than lower scores. It is important that categories are scored appropriately, so 'risk-based' or 'negative' criteria should have low scores if the risk is high to reduce their contribution to the utility (for example, if the cost is high then the score for that option should be low);
- ▲ **Number to be chosen:** This tells the algorithm how many items to include in the selection.

In Portfolio Picker, items in a category are *indistinguishable*, that is, if there are two items available in category *A* picking the first one is exactly the same as picking the second one. Criteria and scoring have a level of subjective input by their very nature. Criteria can be defined from known sources or results, but also from the users own judgement on what is important to the decision being made. Scoring can be difficult, but a good approach involves choosing an appropriate scale for each criterion and being consistent with the scoring process.

Balancing strategies help provide a more balanced selection, and are discussed more in Section 3.

3 The Mathematics behind Portfolio Picker

This section details the mathematics that the algorithms used in Portfolio Picker are based on.

3.1 The Optimisation Calculation

In general, if there are N (a user-defined, fixed value that does not change for a particular portfolio) objects to be chosen from C different categories of object, and there are E different criteria then the overall (raw) utility function U_o for the portfolio can be written:

$$U_o = \sum_{e=1}^E I_e U_e ;$$

$$U_e = \sum_{c=1}^C N_c S_{ce} ;$$

$$N = \sum_{c=1}^C N_c ;$$

where $I_e (> 0 \forall e)$ is the importance level for criterion e , S_{ce} is the score for category c and criterion e , and N_c is the number of objects selected from category c . The idea is to then select N_c to maximise the overall utility. For every possible selection of objects constituting a portfolio, a utility value is calculated. The portfolio(s) with the highest utility is (are) then the optimal choice. Portfolio Picker evaluates a utility value for each possible selection, and the selections with the largest utility are the ones reported.



The way in which Portfolio Picker displays the optimal selections has been updated in Portfolio Picker 1.1. Now, all optimal selections are stored and can be cycled through on the output page. The default selection shown is the one shown in Portfolio Picker 1.0, that is, the selection with the most objects in 'earlier' categories (as defined by the order the categories are given within the software).

To take into account the selection as a whole, a balance penalty function, P , can be introduced into the utility function to give a final utility,

$$U = U_o P$$

where P can take a number of forms ($P = 1$ if there is no balance penalty). This is only one approach amongst many, and there is no generally accepted method that can be applied across a wide range of applications (Weistroffer and Smith, 2005). The two-

stage approach taken in Portfolio Picker has some similarities to the problems considered in Polyashuk (2005).

The “Minimum Utility balancing strategy” uses a Heaviside function:

$$P = \prod_{e=1}^E H(U_e - U_e^{\min});$$

$$U_e^{\min} = I_e \frac{U_0}{(\sum_{e=1}^E I_e)^2}.$$

Selections that give insufficient consideration to one or more criteria are considered inadmissible and are therefore excluded from the optimisation.

A smoother balance function, the “Continuous Penalty Function balancing strategy”, has the following form:

$$P = G(P_e);$$

$$P_e = F\left(\frac{U_e}{U_e^t}\right);$$

$$U_e^t = I_e \frac{U_0}{\sum_{e=1}^E I_e^2}.$$

The choices for F and G (where α determines how rapidly F drops away from unity) are,

$$F(x) = \min(1, x^\alpha);$$

$$G = \prod_{e=1}^E P_e \quad \text{or} \quad G = \min_e(P_e).$$

When $\alpha = 1$ we obtain a linear ramp, but $\alpha < 1$ is also allowed. The two options for G provide two alternative balancing functions, namely a product ramp and a minimum ramp function (respectively).

3.2 The Analytical Hierarchy Process (AHP) Method

Assigning importance values to criteria, especially when there are multiple criteria for the given selection, can be difficult. The AHP method described by Dodgson et al. (2002) below provides a systematic procedure for comparing criteria to obtain a consistent assessment of importance. Its application to the determination of importance values within Portfolio Picker is described below.

For the user, the process involves comparing each criterion with each other criterion, defining which criteria are more important than others. A numerical value indicating which criterion is more important is used to represent the level of comparison. These

numerical values are often based on an odd-numbered scale, for example a 5 point scale:

- ▲ Criterion A and criterion B are equally important: Score 1
- ▲ Criterion A is more important than criterion B: Score 3
- ▲ Criterion A is much more important than criterion B: Score 5

If criterion A is less important than criterion B, then the reciprocal value is used (for example, if criterion A is much less important than criterion B, the score would be 1/5). The user must compare each pair of criteria until a matrix of comparisons has been completed.

Then, the rest of the process involves calculation: The importance values are calculated by taking the geometric mean of each row, and scaling to a standardised scale (0 to 100), then rounding to an appropriate integer value. For example, a coarse scale may have intervals of 10 between each importance value, so the values would be rounded to the nearest 10.



The user can now choose between three different resolution values for the importance values: Coarse (a spacing of 10 between allowed values); Standard (a spacing of 5 between allowed values); and Fine (a spacing of 1 between allowed values).

An example of the process is shown schematically in Table 1 with numerical values used to compare criteria and a standard coarseness for the scale. The equivalent completed AHP screen from Portfolio Picker 1.1 is shown in Figure 1, and the result of calculating the importance values using the AHP method in Figure 2.

It is noted that the diagonal of the matrix in Table 1 contains all 1s (criterion A is equally important to criterion A), and this is why the diagonal cells in Figure 1 are not enabled. Also, only the 'top triangle' of the matrix needs to be specified; the 'bottom triangle' then takes the reciprocal value of the cell in its transposed position.

Table 1: Example of the Analytical Hierarchy Process

	C1	C2	C3	C4	Geometrical Mean	Conversion factor	Final importance value
C1	1	5	3	1	1.97	The upper value of the standardised scale divided by the maximum geometrical mean (50.81)	100
C2	1/5	1	1/5	3	0.59		30
C3	1/3	5	1	1/3	0.86		45
C4	1	1/3	3	1	1.00		50

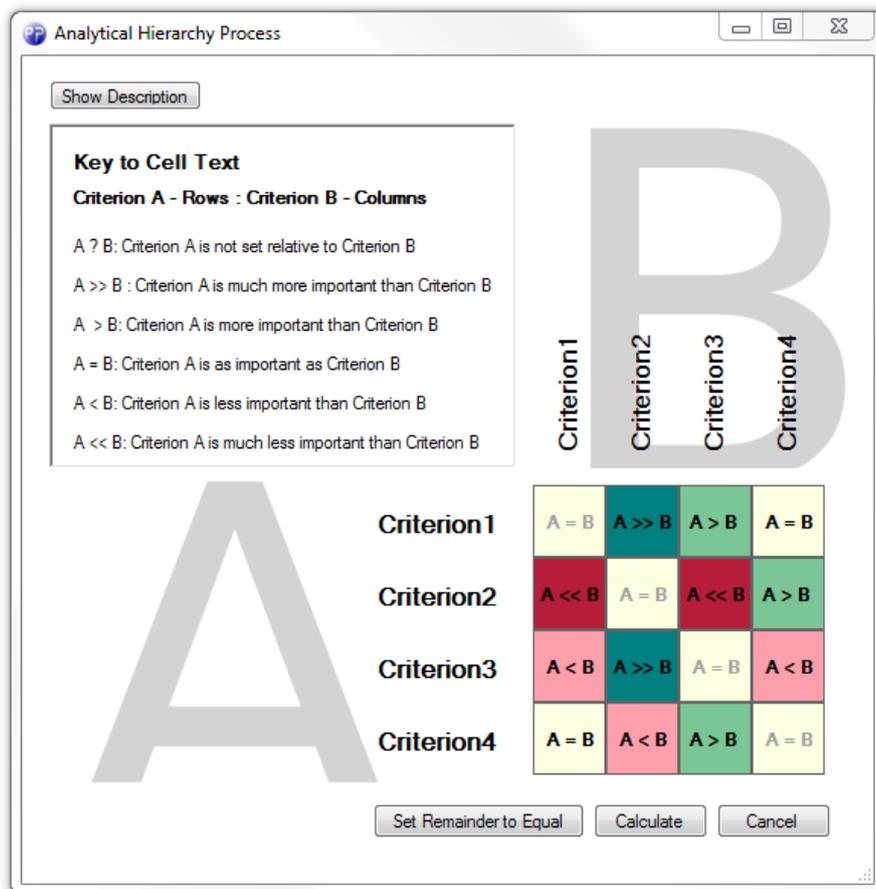


Figure 1: The completed AHP screen corresponding to the example in Table 1



Figure 2: The result from calculating, using the complete screen in Figure 1

References

Dodgson J, Spackman M, Pearman A D and Phillips L D (2000). *Multi-criteria Analysis: a Manual*, Department of the Environment, Transport and Regions, London.

Polyashuk M V (2005). A Formulation of Portfolio Selection Problem with Multiple Criteria. *J. Multi-Crit. Decis. Anal.* 13, 135-145.

Weistroffer H R and Smith C H (2005). *Decision Support for Portfolio Problems*. Proceedings of the 2005 Southern Association of Information Systems Conference.

Appendix A – Glossary of Terms

A glossary of terms used throughout this document is given in Table 2.

Table 2: Glossary of Technical Terms

Term	Description
Object	<p>A single item, from a single category, to be included in the portfolio.</p> <p>An object cannot be in more than one category.</p> <p>Objects cannot overlap (i.e. they cannot have more than one property in version 1 of the software).</p>
Category	<p>A group of objects with a certain property.</p> <p>Categories must be given a number of objects available to choose from.</p>
Criterion	A type of information of interest.
Portfolio/Selection	A selection of objects that gives an overall utility.
Utility function	A mathematical concept that determines the amount of 'information' that can be obtained from a given selection.
Raw utility	The value of the utility function before any penalties or optimisations have been applied.
Penalty function	Used as a multiplier to the raw utility to balance the selection.
Importance	A value linked to each criterion that signifies the relative importance that criterion has for the selection.
Score	A value linked to each category-criterion pair that indicates how well each category meets that criterion.
Sensitivity study	Plots the effect of changing the relative importance values for each criterion (in version 1.0 of the software).
Optimisation	The process of improving the raw utility to provide a more balanced selection.